A Framework for a Critical History of Logic

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The view that science has evolved while using the critical method (dialectics) is undisputed these days. Initially, the traditional view that scientific knowledge is sound and unshakable knowledge, has hindered the view of its development as a critical process. In this respect the history of logic suffered more than other branches of knowledge because the view of logic as developing belongs essentially to the last 150 years. Even today, there is no critical history of logic (the telling of its development by the critical method). This paper is a preliminary attempt in this direction.

1. Preliminary: Critical Standards for Novelty

What is intellectual novelty? To begin with I suggest that we follow Socrates in viewing intellectual activity as fundamentally problem oriented. I therefore use some of the basic assumptions of the theory of questions and answers. This theory has been sometimes tagged “the logic of questions and answers”, but the term “logic”, here is over-pretentious and better left out: the quest for truth is not and cannot be a rigorous full-fledged logic. I therefore refrain from calling it such. I use here the words “problem” and “question” as synonymous

A problem is a set of possible, mutually exclusive answers (or solutions) plus the request/demand for a (tentative) choice of one of them as putatively true. A problem, then, is fundamentally an exclusive disjunction, a set of alternative options plus the request/demand

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1 The use of the terms “problem” and “question” as synonyms is common but far from trivial. I believe there are very good reasons not to endorse it in all contexts. In particular, problems may be tasks of wider ranges, or they may arise when their articulation as questions fail. This, however, should not concern us here.
for choice. A problem is clear when the alternative options it sets are clear and it could be vague to various degrees. The simplest examples for the clearest problems are the many computer programs which are in use by booking agencies. If all dialectical contexts were as simple as these programs (or as simple as these programs in principle), then the theory of questions and answers could indeed by made a full fledged logic. Sylvain Bromberger has pointed out the staggering fact that questions that are too vague to be stated as sets of alternative options are often still clear enough to enable us to recognize whether or not a given new proposition constitutes a possible answer to them. Thus problems, even when vague, often presuppose tacit standards for their solutions. These tacit standards are partial of course, since we are speaking of problems only partly understood. The existence of problems which are only partially understood is in itself no more problematic than the existence of fully understandable problems; what Bromberger draws our attention to is the prevalence of barely understood and even barely understandable problems within research.

Let us then consider the following plain, perhaps even crude dialectical characterizations of novelty.

1. Novelty could be a refutation of a classical solution to a given problem.

2. Novelty could be a discovery of an altogether new solution i.e., a broadening of a problem by means of widening of the classical set of alternatives that we used to identify with the problem.

3. Novelty could be a sharpening of a problem, i.e., the rendering more precise of a set of presuppositions on which the problem rests. This includes making a question less vague by making the standards for being a possible solution to it or the standards of being a true solution to it sharper, or by the refinement of the set of options.

4. Novelty could be a refutation of a problem; a problem can be refuted either directly, by refuting the exclusive disjunction identical with it, or indirectly, by refuting the set of
presuppositions on which it rests. (These two alternatives are logically equivalent but methodologically different.)

5. Novelty could be the replacement of a problem or a set of problems by an altogether new problem or a set of problems, possibly one that partially overlaps with it, possibly even identical with it, but in a completely different framework. (For example, the purpose of the invention of analytic geometry was to replace geometric problems with the equivalent algebraic ones, on the understanding -- possibly erroneous -- that the latter may be solved by the use of algorithms).

6. Finally, novelty could be a new metaphysics. It may suggest new hierarchies of problems, new standards of identifying problems, new standards of deciding solutions, and so on. A change of metaphysics may then be a real upheaval, something that is crudely called a change of paradigm. When it develops sufficiently at a certain domain, it becomes identical in that domain with the change mentioned above, under point 5. If we consider a research program to be a reasonably coherent set of questions, then we may see some research programs as (historically) intensifications of parts of some metaphysical pictures (and not identical with them, as Imre Lakatos maintained).

The parameters are not exclusive since the choice of a mapping of novelties onto parameters is in itself a relatively free decision. A novelty could be easily viewed as of type x, or of type y, or of both. For example, Albert Einstein’s suggestion that space is non-Euclidian is clearly a novelty of type 2, namely a new solution to an old problem, and of type 6, namely a new metaphysics of space pregnant with implications for diverse problems within physics. On the other hand, a novelty could be easily viewed as of type x, or of type y, but not of both. Thus, under one reading of Einstein’s theory of space it is a novelty of type 2, namely merely a new solution to an old problem, but not a metaphysical upheaval. This is Sir Arthur Stanley Eddington’s reading of Einstein’s theory in a quasi-instrumentalist manner. On the other
hand, it may be seen as a real metaphysical upheaval but not as a really new solution to an old problem since the general metaphysics of space that Einstein acknowledged to Leibniz is already non-Euclidian, he said. If so, then viewing it as no upheaval makes it into a novelty of type 3, namely as a sharpening of a problem by means of the sharpening its solutions.

All this is very schematic. It could easily be developed and improved but it suffices for the present purpose as it stands. The exercise here is preliminary. My aim is to test the framework. If successful, such an exercise would simplify the multifaceted task of understanding intellectual progress. Novelties in science, then, should be better delineated and better understood.

2. Major and minor roles of novelties.

The launching of modern logic will be the test case for the present framework. It is too vast and intricate a process to analyze, and mapping the novelties which comprise it is an incredibly delicate task. Consider the following list of novelties: the discovery of the universe of a statement (Augustus De Morgan), its refinement known today as the universe of discourse (George Boole), the discovery of the empty set (Boole), the discovery of the complement set (De Morgan and Boole), the discovery of the algebra of logic (Boole), its refinement into the formal system that we know today as “Boolean logic” (Stanley Jevons, Ernst Schroeder, Edward Huntington), the discovery of modern propositional calculus (Bernhard Bolzano and Boole -- in a rather crude fashion -- then Gottlob Frege, Charles Sanders Peirce and Bertrand Russell), the discovery of the logic of relations (De Morgan, then Frege, and finally Russell), the discovery of the quantifiers (Frege), the discovery of purely formal paradoxes (Russell), formulating a full fledged proof theory (David Hilbert, Kurt Goedel, Alfred Tarski), Goedel’s discovery of his theorems, and their improvement (Goedel, Alonzo Church, Alan Turing, Stephen Kleene), formulating a full fledged theory of
inference (*Tarski, Gerhard Gentzen, Karl Popper*).

These novelties are all very significant. And each is pregnant with profound consequences. Here all of them play “minor” roles, and for the following reason. When they are displayed by means of the framework used here, they will be found to be novelties of type 1-4, but not of type 5-6. The roles that they have played in the history of logic will not be considered as “major”: they will not be considered as shifts of intellectual frameworks or as metaphysical upheavals. Major novelties are elusive, but their influence is substantial and encompassing. Indeed if our framework is successful, it should show that the significance of all novelties, however significant or insignificant in themselves, are seen as such due to a few innovations, ones that we see as the forgings of new frameworks. Let us consider a few examples.

*De Morgan* and *Boole* set formal mathematics (rather then Aristotelian biology) as the model for logic. This was a real (metaphysical) upheaval pertaining to our very understanding of logic\(^2\). It had a tremendous impact on the development of logic, far and beyond any technical innovation that they introduced. Indeed without it, all the novelties mentioned above (technical and non-technical alike) would not have taken place or they would not have been noticed\(^3\). Accordingly its role will be viewed here as major (of type 6).

Or consider the following case: *Frege* is often mentioned as the father of modern logic since he discovered the quantifiers (this is usually presented as a novelty of type 2. i.e. as a new solution to the problem of expressing complex sentences, such as those expressing multiple generality). Quantifiers are of course the single most important technical development of modern logic. But their roles are taken here to be minor all the same. In his

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\(^2\) This point is discussed in detail in my: Logic between Methodology and Epistemology. Iyyun, forthcoming.

yet to be published) book on *Ludwig Wittgenstein*, Joseph Agassi claims that Frege’s most important contribution was a shift of agenda: he was the first great logician who gave up (in a clear and explicit manner) the task of securing natural science on a solid footing by means of logic. Frege’s greatness, then, lies first and foremost in his major novelty, the meta-logical shift of agenda which he initiated: his abandonment of a research program which had predominated logic for over 2000 years, and its replacement with a new one – his logicism – which was both more promising and more humble (and only later discovered to be yet too ambitious).

The major novelty that will engage us here is this: traditionally logicians studied judgments and sound inferences, yet somewhere along the way, between Hume and Tarski, they broadened their subject matter considerably and began to study propositions and valid inferences as such. They no longer viewed it essential to concentrate on valid inferences with judgments as their premises, but rather they studied all valid inference (with or without premises, true, false, and contingent)

At first sight this novelty can be taken to be a type 3 minor (even technical) novelty: Obviously it seems to be a sharpening of a traditional task. Yet it is much more than just that. It is a major novelty of type 6, since it is a major change of understanding of the logician’s place and task: modern logicians, qua logicians, are indifferent to epistemology. Allow me a few words of explanation.

In hindsight, it may be noted that judgments were illusory life-lines, symptoms of the decay of traditional logic. Traditional logicians did not care much for tautologies (they viewed them as empty, vacuous, useless truths). They cared for scientific knowledge, that is knowledge that they deemed informative and certain. Until Frege’s time, then, logicians sought necessary informative truths by means of logic (classical or other). This search was deemed essentially complete. So traditional logicians were committing themselves to the view
that logic discusses informative propositions that are known to be true. It was absurd (yet it took years of laborious work to realize it as such). And until it was realized as such the theory of judgment was called for, as a life-line. It was a silent means to conflate logic (the theory of valid inference) and the ideal of science as informative knowledge, i.e., as a set of all known informative sound inferences. Thus, the theory of judgment traditionally included ontological presuppositions that insured that the scientific picture of the world is right (made of specified particulars that bear specified essences etc.), epistemological presuppositions that allegedly insured the truth of the ontological presuppositions, and even psychological presuppositions that allegedly insured the truth of the epistemological ones. It was a house of cards.

In the next section I describe its collapse, by means of my framework. Here I only wish to emphasis that that the theory of judgment imposed constraints on logic, and that the freedom from these constraints on logic is indispensable: without it logic still wavers between tradition and modernity. Surprisingly, this might be the case of Frege. He rejected the discovery of the primacy of propositions over judgments on some defunct meta-logical grounds: amazingly, he viewed as useless all inferences from false premises or even from mere conjectures⁴. This is especially odd, since, as we have noted, he was the first logician to construct formal logic after having given up the task of securing empirical science by its means. So our framework may help to discern his sitting on the fence. On the one hand, he was the first modern logician, since he was the first to give up the logician’s concern for epistemology. On the other hand, he still clung to judgments, as something without which the logicians work is vacuous, and useless. He ignored the fact that without the concern for epistemology the logician had no need for the theory of judgment.

⁴ All relevant references have been ably collected and commented upon in Gordon P. Baker, & Peter M. S. Hacker: Frege: Logical Excavations. Oxford 1984, pp. 37-39. Of particular interest are the letters Frege wrote to Hugo Dingler and to Philip E. B. Jourdain. They are to be found in Gottlob Frege: Philosophical and Mathematical Correspondence. Oxford 1981.
3. The neglect of judgment

In this section, I wish to characterize the time of the grandparents of modern logic as one of tremendous pressure, on some of the greatest thinkers who ever lived, to solve an urgent but insoluble problem. It was insoluble, as it comprised a poor research program. The research program reflected a poor metaphysics. I identify the origin of modern logic as the initiation of a new metaphysics that offered a better background and a more productive new set of problems to be solved.

The problem was the so-called ‘epistemological problem’: How are we to prove informative theoretical knowledge? It was felt that if such a problem is insoluble – if informative theoretical knowledge cannot be proved – then, the very foundation of science, the most solid of human achievements, is shaky. This was the cause of the urgency and the source of the poverty of the research program. To admit that science is conjectural seemed a disaster for reason. Hence, the tremendous pressure to find the solution. It is easy to notice in retrospect that not only was the epistemological problem insoluble, as Hume had already noted, but also the very conditions for a satisfactory proof of any assertion were poorly understood. They were somewhat poorly understood until the 30’s of the 20th century if not later.

Great pressure often invites extreme efforts that lead to extreme results: either exceptionally ingenious and innovative or exceptionally traditional and defensive (more often the latter than the former). This is true of logic as it is true of other fields. Among the exceptionally ingenious contributions to logic resulting from the pressure to solve the epistemological problem stand some of the most eminent and valuable failures of human thought to secure reason on solid footing (including Gottfried Wilhelm Leibniz and Immanuel Kant). Among the exceptionally traditional philosophies of logic, let us mention here
scholasticism (whose culmination is perhaps best represented by the –Post-scholastic –
celebrated Port-Royal logic of Antoine Arnauld and Pierre Nicole) and German Hegelianism
(which, in commending inconsistency, could not produce any noteworthy output within the
theory of logic).

Bolzano and Boole (independently, a decade or so apart) unintentionally showed the
way to put an end to this unfeasible race for certainty in most remarkable ways that were
simultaneously both strikingly traditional and innovative. They were the very last great
logicians to have shared the old research program that aimed at solving the epistemological
problem primarily by the study of logic. They did not try to remove the quest for certitude.
Yet, their problematic, inadequate solutions suggested a giant leap forward. They refuted
classical solutions to classical problems of logical theory; they sharpened classical problems,
expanded others, and even refuted the presuppositions behind some problems — by replacing
them with new ones. Finally, they suggested a new hierarchy of meta-problems in philosophy,
setting, for the first time, pure logic as the foundation of mathematics and as independent of
biology and metaphysics. They imposed on those who followed them a new metaphysics that
they themselves were not yet ready to accept without reservations. They are the first parents
of modern logic and more.

Here is an example of a refutation of a classical problem that they initiated. Consider
the classical problem of analyticity: What judgment is analytic? Today when we are used to
formal logic so much, we easily forget all about judgment and ignore the enormous role it
played in history. It’s role was very special though: it was to smooth over the difficulty (and
in retrospect, the failure) to solve the epistemological problem. The theory of judgment
constituted a part of traditional logic and it warranted the maintenance of their subject matter
as comprising not only necessary truths devoid of informative content (tautologies) but as
including also necessary informative truths. This is the secret of “conflated definitions” which
historically defused into “judgments”. I wish to briefly expose it now.

The secret was a traditional conflation of two distinct senses in which definitions may serve: they may be nominal definitions i.e. define the meaning of words or introduce new words using old ones, or they may be real definitions, that is they may actually embed conjectures as to what there is. The chimerical “conflated definitions” which later defused into the theory of judgment, were allegedly both logically necessary (nominal) and informative (real). Let me explain.

Ancient logic constituted an important, integrated part of traditional epistemology. It embedded, putatively, a set of techniques that helped, putatively, to find true judgments (scientific, proven, rational truths). It also embedded, putatively, a set of techniques which helped, putatively, to find and distinguish necessary truths from merely contingent ones. The set of techniques required here are clearly distinct: telling truth from falsehood and necessary from contingent; yet traditionally they were conflated. Regrettably, the classical theory of definition, the classical theory of intuition and even the classical theory of induction, were suggested (already by Aristotle) as tools for the differentiation between the true and the false as well as between the necessary and the contingent.

Here is the point. The best known expression of this conflation is “existential import” so-called. Traditional logicians used it readily and modern logicians proscribe it. It was far less innocent than it is usually presented in logic courses: it was the bridge between nominal definitions and real ones, as it warranted the move from nominal to real definitions (and thus from semantics to ontology). If we agree to define Man as “rational Animal” we are in the field of semantics. If we allow existential import, our definition can be real and it leads to the field of ontology.

The theory of judgment though embedding this very conflation -- ever since the birth of logic -- was readily received as it, putatively, helped get rid of the oddities described above,
and more. In particular, it helped to put off the concern that the conflated definitions, later called “judgments” were either not really necessary truths, or not informative. Thus, for example, Aristotle’s syllogism is never presented by himself as a theory of valid inference but rather as a theory of valid inference from necessary definitions (such as, the definition of man as a rational animal).

Benedict de Spinoza noted an absurdity within the traditional conflation. He said, when all is said and done, all truths are necessary in the eyes of God, from the viewpoint of Eternity. Thus, the attempt to distinguish between the contingent and the necessary is a sign of human (all too human) ignorance. This is an amazing observation. It justly caught Leibniz’s attention. He sensed an outrage, as he rightly understood this to imply Spinoza’s determinism. Determinism, many people suggested, threatens the very foundations of Christian morality: moral choices must be contingent if there is free will. When all is said and done, said Leibniz, humans must be punished for sinful acts only if they are products of choices and not otherwise. Leibniz wanted free will, but he also wanted all truths to be necessary in the eye of God. This is a central logico-theological puzzle. He invested extraordinary ingenuity in an attempt to solve it. He sought a logical criterion for his distinction between necessary and contingent truths. He deemed analytic all truths, be they necessary or contingent. So his effort was to distinguish between two types of analyticity: necessary and contingent, and to offer a criterion for this distinction.

We need not analyze here his extraordinary attempts to provide such an extraordinary criterion. In retrospect, it is easy to point out the source to the difficulties he faced: Ancient logic dealt almost exclusively with informative truths introduced as conflated definitions considered judgments. Some informative truths were deemed necessary (“All humans are Rational”), others were deemed accidental or contingent (“All humans are featherless bipeds”). In fact, it was impossible to distinguish the two groups by mere logical means
because both were sets of informative contingent conjectures. (Moreover, “All humans are rational” and “All humans are featherless bipeds” happen to be false). This fact portrays the important aspect of ancient logic which is our topic: it aimed to prove the unprovable, and thus invited the theory of judgment to smooth over the difficulty that counter-examples raise: judgment rubs them out. The difficulty is not with the counter-examples, however, but with the overpowering means used to avoid them, namely, the circularity that judgment incurs. In fact, judgment is question-begging as it tells us what we anyway consider true.

In retrospect, it is easy to see that already Hume noticed this. He observed that only non-informative truths (now called “tautologies”) are logically provable. This simple observation is probably the greatest single shock modern philosophy has known. It was scarcely noted at the time, however, since I presume, Hume’s philosophy was shocking in ever so many respects. It still never ceases to amaze that this indeed is the case: Hume’s rather trivial observation – informative theoretical truths cannot be proved – was the greatest challenge modern philosophy has faced. We should remember that even Russell was stunned to discover it. I mention this here because it highlights an interesting fact that hitherto was ignored: Hume is the first grandparent proper of modern logic proper. He was the first to anticipate the scope and limits of this yet-to-emerge new field, the first to sense, by a mixture of insight and foresight, logic and perspicuity, that a theory of proof would produce tautologies, not judgments (informative, necessary).

Modern logic was then still a very distant image far in the future. Before its birth, the best minds of the modern era would awaken from their dogmatic slumber and attempt to face the challenge. They would attempt to annex to the realm of certainty the contingent in addition to the necessary (Leibniz) and some of the synthetic in addition to the analytic (Kant). Indeed, Kant designed his transcendental logic as the grand plan to secure informative theoretical knowledge. Today we notice with surprise that transcendental logic was never a
logic proper, nor was it ever presented as logic proper: it was never more than a set of propositions; it was never more than a tag-name for an aspiration, an aspiration for axioms and derivation rules that were never worded, yet that allegedly already proved the unprovable.

Traditionally, logic was considered as the theory of judgments, then, not of propositions. Its very usefulness seems to have been dependent on the ability to tell a true proposition, a proper judgment, from mere conjectures, not to mention falsehoods. Then came Bolzano and William Whewell and Boole and refuted the classical problem (what is an analytic judgment?) Let us observe how.

As with many problems the problem we discuss now – what is analytic judgment -- has a somewhat vague pre-history. Aristotle did not pose it explicitly. Had he posed it, his answer would have been somewhat vague as he (and more so his disciples) resorted to the problematic notion of ‘primary definitions’. The status of primary definitions is unclear; it is clear that primacy and analyticity are not identical, but it is not clear that Aristotle would have agreed to this. For example, he said, rightly of course, that primacy is non-hereditary. And, we all agree, analyticity clearly is: a judgment derived from an analytic judgment, is analytic, yet primacy as Aristotle understood it, whatever exactly it is, is certainly lost even after the first and the most trite logical move. Consider the assertion “all humans are rational animals”. If it is analytic, then so is “all non-rational animals are non-human”, since, as Aristotle has taught us, they are logically equivalent. Nevertheless, traditional logicians took it as a matter of course that the former is primary whereas the latter is not.

Arnauld seems to have first provided a straightforward Aristotelian idea about analyticity: a judgment is analytic whenever it expresses a necessary, true assertion accurately describing an essence. This is true but not helpful at all, as there is no difference that we know of between the two concepts: Arnauld still had the initial need to explain why analyticity is hereditary, and primacy is not hereditary.
Leibniz disagreed. He maintained that all truths (necessary and contingent, primary or not, concerning essences or accidents) express really analytic judgments. All truths, he said, are reducible by this or that valid transformation to the single identity statement $A = A$. Hence, they are all analytic. He added that at most we will have a difficulty to see that, because the number of these transformations may increase indefinitely. We may then take resort to ‘the principle of sufficient reason’ and a proof machine labeled ‘Characterisitca Universalis’ in order to prove the analyticity of any given judgment. But, largely, he concluded, the undertaking is possible.

Kant disagreed. He divided all necessary definitions into analytic and synthetic and claimed analytic judgments comprise only a small part of the sum of provable judgments. He titled ‘transcendental logic’ the set of all provable universal statements.

These then are the three classical answers to our question. Alberto Coffa has suggested to neatly arrange them a-historically as a gradual narrowing down of the domain of analyticity: by Leibniz all true judgments are analytic, by Arnauld true judgments that are necessary are analytic, and by Kant only those true necessary judgments that are not transcendental, are analytic.

Let me show now that the question was refuted, and that the refutation was the discovery that analyticity is a property of propositions, not of judgments. Logic should be the discussion not of judgments but of propositions. This is an obvious shift of agenda. Who made this remarkable discovery in an unequivocal explicit manner? It seems no one did before Russell. (Frege’s odd refusal to acknowledge the validity of inferences from falsehoods and even from conjectures attest to this amazing delay in the recognition of the expulsion of judgment from the domain of formal logic. See note 4.)

The shift was effected when Bolzano split the classical problem (What judgment is analytic?) He split it into 1. What proposition is analytic?
and 2. What makes the truth of a proposition analytic?\(^5\) This distinction is remarkable since obviously the class of analytic propositions includes only true ones. He also distinguished these two questions from the psychological question, What is an act of assertion? Either questions make no reference to judgment, since it was Bolzano’s point that analyticity is determined regardless of whether anyone asserts it as true.

Whewell intended a parallel move. He failed, however. Under Kant’s influence, he divided all truths into necessary and contingent -- in a manner that seemed to suggest that he would follow the same route as Bolzano\(^6\). He recognized the validity of Hume’s critique of induction (ibid., 72). He thus paved the way to the modern understanding of logic as the study of truths devoid of informative content. He admitted only necessary truths as within the realm of logic. Like Hume, he should have admitted only nominal definitions as necessary. Yet, he aimed at refuting Hume by showing that the natural sciences are a set of truths which are both necessary and informative. He realized that nominal definitions are not informative. So he set out to show that definitions in science are in a sense nominal and in a sense real. Thus, in his view, definitions in science are necessary and yet informative. Consider, for example, the axioms of Newtonian mechanics. Whewell considered them nominal definitions of the concepts that he labeled “apriori concepts”. He explained this term thus. Apriori concepts are necessary (transcendental) intellectual constructs. In a sense, they are real and informative since their discovery is a historical event, and since they apply to reality and their application is universal: they enable the anticipation with certainty of some future events. This is an obvious case of what I described above as conflated definition. I mention it here in order to illustrate the subtlety of the theory of judgment: it is diffused into the theory of science. It does so even when modern logic was budding. Today it is easy to see why Whewell failed:

\(^5\) This is a major point in his monumental Wissenschaftslehre of 1837. See: Bernard Bolzano: Theory of Science Oxford 1972. Surprisingly, however, the first to clearly note this in relevance to our context was Alberto Coffa in his: The Semantic Tradition from Kant to Carnap: To the Vienna Station. Cambridge 1991, Chapters 1-2.

nominal definitions cannot be refuted, yet if they are taken in some sense as real, they are -- in that same sense -- not necessary, not apriori. Often they are refutable (refutable in principle that is, since the truth cannot be refuted in fact). This, then, was the very last instance of this conflation in the writings of a pioneer.

*Whewell* influenced *Boole* greatly. Yet *Boole’s* algebra of logic is a straightforward theory of terms and propositions, with no mention of judgments. It suggests explicitly a division of all truths into logical truths (tautologies) on the one hand, and contingent ones (empirical, informative) on the other – with no mention of knowledge. (*Boole* easily made the distinction, not worrying about securing knowledge, as he believed that he could secure this later on with the aid of his theory of probability). So he did not hesitate to call all informative truths contingent. Only tautologies are true in all universes of discourse and are hence analytic. Contingent propositions are only true in some universes of discourse and are hence synthetic. (He believed some synthetic truths are highly probable yet contingent all the same.) Thus, he excluded all ‘real definitions’ and all conflated definitions by fiat. He is the first to have refuted in a clear manner the classical problem of analytic judgments (refuted in the sense explained above as a novelty of type 4).

Like *Whewell*, *Frege* followed both *Hume* and *Kant*. Following *Hume’s* critique of induction he suggested that at least for the time being, logicians qua logicians should abandon epistemology, and study only truths devoid of informative content. But following *Kant* he could not give up the view of logic as a theory of sound inferences. He could not endorse the classical view of judgment. He stuck to judgments nonetheless as needed for the theory of sound inference, yet in a new sense of the term, which was aimed at a new compromise between the nominal and the real senses in which judgments are true. Indeed, his life’s project was based on the assumption that despite *Kant*, arithmetical propositions are analytic, yet

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7 This point is already evident in his trail blazing *George Boole*: The Mathematical Analysis of Logic. Bristol [1847] 1998.
despite their analyticity (the fact that they all share the same reference) they do not share a sense. Thus, in his view, arithmetical judgments are necessary and yet informative (in new and weaker sense).

The final transition from judgment to proposition, then, clearly adumbrated by Bolzano, repeated somewhat ambivalently by Whewell, was entrenched as part of logic proper in Boolean logic despite some Fregean last ditch efforts. Boolean logic is so amazing just because it fully ignores judgments. As to Russell, clearly, his need to resolve his own paradox by declaring it meaningless was the watershed, as it made clear for the first time the need to distinguish clearly between a proposition and a judgment and the indispensability of recognizing contradictions as propositions proper, as an integral part of language. Thus, modern logic was in full swing.

Let us sum up then: At the heart of traditional logic lies the traditional question, what is a sound inference? Modern logic took its first step by criticizing this question (Hume, Bolzano and Boole) and, by replacing it with the modern question, what is a valid inference? (Boole, Russell and Tarski). This was not a technical novelty. It was a type 6 novelty, an upheaval, a meta-logical revolution which determined future agenda for years to come. Key figures, such as Whewell and Frege, did not take part in this revolution. This discovery is an interesting result of the use of our framework: Their achievements were made despite them being fence-sitters. On the one hand, they still clung to judgments, viewing as useless the merely valid inferences. On the other hand, they postponed the task of securing empirical science by means of logic.

Summary:

Im Mittelpunkt der traditionellen Logik steht die traditionelle Frage: Was ist eine begründete Folgerung? Die moderne Logik tat ihre ersten Schritte mit der Kritik dieser Frage (Hume, Bolzano und Boole) und mit dem Ersatz dieser Frage durch die
moderne Frage: Was ist eine berechtigte Folgerung? (Boole, Russell und Tarski).

Beträchtlicher Fortschritt wurde auch von jenen gemacht, die sich nicht für eine der zwei Fragestellungen entscheiden konnten (Whewell, Frege).

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